Economic growth with hysteresis

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Contents

Standard growth theories

- The analytical methods used in growth theory usually involve conservation and reversibility properties.
  - Economic shocks will then have no lasting deleterious effects on potential output, despite of the debilitating effects of recessions on bankrupted firms and unemployed workers, and
  - booms have no long-lasting beneficial effects on potential output, despite the new capital investments made and learning-by-working of employees.
- In contrast, I am especially interested in whether temporary shocks have permanent effects on (sustainable) economic growth.
- This property – temporary shocks having long lasting traces – is called *hysteresis*.

Asymmetric response to shocks

- Because an economy is not a closed system, it faces shocks from outside.
- The way the economy responds to these shocks depends on the structure of that economy.
- In an economy, a negative shock has in general a stronger impact on the development pattern of the economy than a positive shock of equal size. Structural reason for this are e.g. the following:
  - When a worker gets unemployed, he loses productivity while unemployed.
  - When a firm gets bankrupt, it loses its intangible firm-specific capital

Hysteresis and public policy

- Governments attempt to stabilize economic development by evening out shocks running a deficit with negative and a surplus with positive shocks.
- However, because of the asymmetric response, economic development is not a sine curve: downward responses to shocks are in general stronger than upward responses.
- As a consequence of this, public debt tend to accumulate: governments fail to pay their debts during negative shocks by surpluses during positive shocks.

Hysteresis as a physical phenomenon

- The concept of hysteresis origins from physics: it occurs in ferromagnetic (or ferroelectric) materials, as well as in the deformation of some materials (such as rubber bands) in response to destabilizing forces.
- Such a system is dependent not only on its current environment but also on its past environment.
- When economic growth performs hysteresis, temporary shocks cause a permanent (or very persistent) effect on the average growth rate of the economy.

Cross

Hysteresis is relevant to economic analysis for the following reasons (cf. Cross et al. 2009):

1. The agents can be confronted with a binary choice (e.g. to enter or not to enter the market), which requires an action almost instantaneously.
2. Switching between states is subject to sunk costs: when an action is taken, an expense is incurred that cannot be recouped on reversing the action. All this creates threshold values of the control parameter when an agent is willing switching from one strategy to another.

Analogy to mechanics

- Many growth models predict that economic fluctuations do not change potential output paths, but the evidence suggests that they change in particular in recessions (Cross et al. 2012).
- Cross et al. (2012) construct a model to explain this phenomenon based on an analogy with water flows in porous media.
- Potential output displays hysteresis with regard to aggregate demand shocks and thus retains a memory of the shocks associated with recessions.

Hysteresis in economics 1

- The concept of hysteresis is used in economics to describe
  - the time path dependence of the “natural” rate of unemployment in labor economics [cf. Sachs (1986), Blanchard and Summers (1986), and Lindbeck and Snower (1986)].
  - the behavior of the foreign exchange rate [cf. Baldwin and Krugman (1989), and Dixit (1989)].
- Turnovsky (2000, chapter 11) examines the effects of temporary public policy on economic dynamics.

Hysteresis in economics 2

- In empirical literature, it has been common to analyze hysteresis by linear difference (or differential) equations with breaks, lags or a unit root [cf. Göcke (2002)].
- In empirics, hysteresis takes the form of a break, a lag or a unit root.
- However, there is so far no model to explain the effects of temporary shocks on the growth rate of the economy.

Conclusion: there is a need of endogenous growth models with hysteresis properties!

Basic model (Lucas 1978, Barro 2009)

- Time is discrete, one consumer, only one good "fruit"
- The number of productive units, "trees", is fixed, i.e. there is neither investment nor depreciation
- The log of output evolves as a random walk with drift:

$$\log y_{t+1} = \log y_t + g + u_{t+1} + v_{t+1},$$

where g is the exogenous growth rate,

u_{t+1} random term normal with mean 0 and variance σ^2. This characterizes normal "symmetric" economic fluctuations.

v_{t+1} random terms that picks up low-probability disasters. In these rare events, output jumps sharply downwards.

The distribution of random events

- The probability of more than one disaster in a period is assumed to be small enough to neglect.
- In a disaster, output contracts by the fraction $b \in (0, 1)$.
- The distribution of v_{t+1} is the following:

$$v_{t+1} = \begin{cases} \log(1-b) & \text{with probability } p, \\ 0 & \text{with probability } 1-p. \end{cases}$$

- The disaster size, $b \in (0, 1)$, can be constant or follow some probability distribution.

Extensions

- Multiple goods: different trees produce different fruits.
  - Households consume all kind of fruits with a Cobb-Douglas utility function
  - Random demand shifts can be introduced on the assumption that the parameters in the Cobb-Douglas utility function are random variables.
- Endogenous growth: some fruits are planted to have more trees in future.
  - This leads to the AK model where output is in fixed proportion to capital (trees).
- Lags in investment: a new tree must grow big enough to produce fruits.

Critics 1

- The fruit-tree model is proper for predicting the behavior of an economy after a random disaster: it models asymmetric behavior exogenously having a random disaster, but not a random windfall.
- That model is as it stands improper for our purposes: to explain why symmetric shocks causes asymmetric behavior.

Critics 2

- There are two sources of asymmetry in the economy: the possibility of a firm to get bankrupted; and/or the possibility of a worker to get involuntarily unemployed.
- There is no “wage labor” and consequently no unemployment.
- There is no bankruptcy, because all capital (fruit trees) is ***tangible*** and can therefore be sold in the market or used as a collateral for loans.

How to model bankruptcy?

- Assume a single producer-owner entrepreneur who produces only from capital.
- ***Inequality constraint***: a firm gets bankrupted, once its tradable assets fall below its debt.
- The key concept is ***firm-specific intangible capital***, which cannot be sold or used as a collateral.
- In order to enter the market, a new firm must invest in firm-specific intangible capital by credit, i.e. a new fruit tree must grow big enough to produce fruits.
- Once the firm faces a big enough negative shock, its inequality constraint becomes effective and it bankrupts loosing all its firm-specific capital.

How to model hysteresis in the labor market?

- Assume that the firm produces from both intangible capital and wage labor.
- At the occurrence of a negative shock the firm lays off workers, and at the occurrence of a positive shock it hires more workers.
- A worker accumulates firm-specific tangible knowledge by learning by doing while he is employed. This knowledge increases his productivity in the workplace.
- When a worker looses a job, he looses his firm-specific knowledge and his productivity falls.

Connections to models of endogenous growth

- Endogenous growth is based on the accumulation of intangible capital that spills over from one firm to another (or from one worker to another).
- Creative destruction: a less-productive tree is replaced by a new more-productive tree. The owner of the less-productive tree loses its assets.

Household

Following the original idea of Klaus Wälde (2002), I have combined capital accumulation and stochastic technological change as follows (Palokangas, CESifo paper 2003):

- The representative household's expected utility is

$$E\int_T^\infty \frac{1}{1-\sigma}[(1-s)y]^{1-\sigma}e^{-\rho(t-T)}dt, \quad \sigma \in (0,1)\cup(1,\infty),$$

where E is the expectation operator, y output, $0 < s < 1$ propensity to save, $(1-s)y$ consumption, $\rho > 0$ the constant rate of time preference, σ the constant rate of risk aversion.

Production

- Efficiency in production is a^γ, where $a>1$ is a constant and γ is the serial number of technology.
- In the advent of technological change,
  - the serial number γ increases by one
  - efficiency increases from a^γ to $a^{\gamma+1}$
  - a fixed proportion δ of existing machinery k is destroyed, because it does not function with the newer technology.
- The household has one unit of labor, of which it devotes l to production and $1-l$ to R&D.
- Given total factor productivity a^γ, output y is produced only from capital k and labor l through technology $f(k,l)$:

$$y = a^\gamma f(k,l).$$

Technological change

- In a small period of time dt, the probability that R&D leads to development of a new technology is given by $\lambda(1-l)dt$, while the probability that R&D remains without success is given by $1-\lambda(1-l)dt$, where $1-l$ is labor devoted to R&D and λ is the productivity of R&D. This defines a Poisson process q with

$$dq = \begin{cases} 1 & \text{with probability } \lambda(1-l)dt, \\ 0 & \text{with probability } 1-\lambda(1-l)dt, \end{cases}$$

where dq is the increment of the process q.

Capital accumulation

- Capital k is a stock of goods that does not depreciate. Because it is the only assets in the model, saving in a small period of time dt equals deterministic capital accumulation, dk^d:

$$dk^d = sy\,dt = sa^\gamma f(k, l)dt.$$

- In addition, in the advent of technological change, capital jumps from k to the new level $\widetilde{k} = (1 - \delta)k$, because the remainder δk becomes obsolete.
- Thus, total capital accumulation is given by

$$dk = dk^d + (\widetilde{k} - k)dq = sy\,dt - \delta k\,dq.$$

Utility maximization

- The household maximizes its expected utility by the saving rate s and the allocation of labor $l \in [0, 1]$ subject to capital accumulation and technological change.
- Define the value function

$$\Gamma(\gamma, k, T) \doteq \max_{s,l} E \int_T^\infty \frac{1}{1-\sigma}[(1-s)y]^{1-\sigma} e^{-\rho(t-T)} dt.$$

Bellman equation

The Bellman equation is

$$\rho\Gamma(\gamma,k,T) = \max_{s,I}\left\{\frac{1}{1-\sigma}[(1-s)y]^{1-\sigma} + \Gamma_k(\gamma,k,T)\frac{dk^d}{dt} + \left[\Gamma(\gamma+1,\widetilde{k},T) - \Gamma(\gamma,k,T)\right]\frac{dq}{dt}\right\}$$

$$= \max_{s,I}\left\{\frac{(1-s)^{1-\sigma}}{1-\sigma}a^{(1-\sigma)\gamma}f(k,I)^{1-\sigma} + \Gamma_k(\gamma,k,T)sa^{\gamma}f(k,I) - \lambda\left[\Gamma(\gamma+1,(1-\delta)k,T) - \Gamma(\gamma,k,T)\right](1-I)\right\}.$$

Solutions

- With Cobb-Douglas technology $f(k, l) = k^{\alpha} l^{1-\alpha}$, both the saving rate s and the proportion of labor devoted to R&D, $1 - l$, are constants.
- This means that the economy is in a stationary state that follows a cycle around the equilibrium growth path.
- With a more complex function (e.g. CES), it is likely that the model can be solved only numerically.

Extensions

The model can in principle be extended into the following directions;

- Instead of one representative agent, there can be a number of producer-consumers that are strategically interlinked. Imperfect competition or spillover of technology between producers create strategic like between these, which establishes a dynamic game.
- The capitalist own firms, but do not work. The workers supply their labor and savings to the capitalist. There can be different forms of wage settlement (e.g. union-firm bargaining, efficiency wages) in the model.