Conceptualising Resilience in a Decision-theoretic Context

Michael Kuhn and Stefan Wrzaczek (IIASA)

Instat Risikokonferenz
Wartburg, 03-05 May 2023
International Institute for Applied Systems Analysis (IIASA)

- Contribute to building sustainable, more resilient and fairer societies/economies
- Lead on applied systems analysis and integrated modelling
- Transform science into policy
- Build capacity to deal with global challenges
- 24 member countries
- ~400 staff (300 researchers)
- ~ 22 Mio EUR balance

Six Research Programs

- Advancing Systems Analysis
- Biodiversity and Natural Resources
- Energy, Climate, Environment
- Economic Frontiers
- Population and Just Societies
- Strategic Initiatives
Economic Frontiers

- What **behavioral changes** are required to achieve **social and environmental transformations**?

- What policies and **institutional reforms** are needed to bring about the **required incentives**?

- What **impact on wellbeing** across **social strata, geographical scales** and **time**?
Introduction

Background:

- Well-known measures of resilience based on eco-systems modelling (Holling 1973).
- Some socio-economic conceptualisations (Keating et al. 2014) but few decision-theoretic formulations to date (Polasky et al. 2011, Li et al. 2017).

Objectives:

- To set out a (simple) model of renewable resource use and conceptualise resilience in a rigorous decision-theoretic way.
- To derive a model-based measure of resilience and apply it to assess resilience of resource use.
Introduction

Background:

- Well-known measures of resilience based on eco-systems modelling (Holling 1973).
- Some socio-economic conceptualisations (Keating et al. 2014) but few decision-theoretic formulations to date (Polasky et al. 2011, Li et al. 2017).

Objectives:

- To set out a (simple) model of renewable resource use and **conceptualise resilience in a rigorous decision-theoretic way**.
- To derive a **model-based measure of resilience** and apply it to assess resilience of resource use.

Analogous reasoning can be applied to corporate decision-making.
Model ingredients

- (Optimal) exploitation of a **renewable resource subject to random shocks**
- (Optimal) behaviour leads to **long-term sustenance** of the resource stock if and only if the level of the stock is above a (Skiba-)threshold.
- **Random shock** may put resource stock below the threshold.
- Appropriate **actions** (e.g., pre-cautionary extraction) allow the decision-maker to increase the probability of remaining above the threshold.
Resource renewal

- Economy in which consumption $C(t)$ is harvested from a renewable resource stock $R(t) \rightarrow$ decision

- Resource dynamics: $\dot{R}(t) = g(R(t)) - C(t)$ with $g(R(t)) = \frac{aR^2}{b + R^2}$ as replenishment $\rightarrow$ state

- Shock arrives at exogenous rate $\eta$ and destroys $D(\tau) = (1 - \epsilon)R(\tau)$ of the stock at random time $\tau$.

- Two stages: 1 = before shock; 2 = after shock
Resource renewal

- Economy in which consumption \( C(t) \) is harvested from a renewable resource stock \( R(t) \) → decision

- Resource dynamics: \( \dot{R}(t) = g(R(t)) - C(t) \) with \( g(R(t)) = \frac{aR^2}{b + R^2} \) as replenishment → state

- Shock arrives at exogenous rate \( \eta \) and destroys \( D(\tau) = (1 - \epsilon)R(\tau) \) of the stock at random time \( \tau \).

- Two stages: 1 = before shock; 2 = after shock

= payouts, dividends in a corporate context

= change in assets

= profit

= devaluation/loss of assets
Convex-concave production

Renewal/revenue $g(R)$

High-state = stable

Low state = unstable

Increasing-then-decreasing returns

- Initial scale returns (fixed baseline costs; learning-by-doing) followed by “overreach”
- Network effects: connectivity vs. crowding

Optimal allocation: Marginal renewal/revenue = discount rate

Resource / asset stock
Convex-concave production: marginal returns

**Marginal renewal/return**

**$g'(R)$**

**Discount rate**

- Low state = unstable
- High state = stable

**Optimal allocation:** Marginal renewal/revenue = discount rate

**Increasing-then-decreasing returns**

- Initial scale returns (fixed baseline costs; learning-by-doing) followed by “overreach”
- Network effects: connectivity vs. crowding

**Resource / asset stock**
Illustrating resilience

Stage 1 resource path with consumption/harvest $C_1$

- Resource stock $R(t)$
- Long-run steady state $\hat{R}_1$
- Threshold level $R^{Skiba}$

Time $t$

$t_0$, $\tau$
Illustrating resilience

Stage 1 resource path with consumption/harvest $C_1$

Resource stock at time of shock – Damage > Threshold

Stage 1 resource path with consumption/harvest $C_1$
Illustrating resilience: Dependence on ex-ante consumption/extraction policy

Greater consumption of the resource leads to loss of resilience.
Illustrating resilience: Dependence on timing

(For declining resource levels) a more consumption-oriented policy may be resilient to early-enough shocks. For increasing resource levels the reverse is true: time to build resilience.
Illustrating resilience: Measuring $R(t)$

Illustration of resilience measurement with $R(t)$, $R_{Skiba}$, and $R_1$. The graph shows two paths:
- Resilient path
- Non-resilient path

**Stage 1 resource path with $C'_1 > C_1$**

- Count all times at shock where the resource level exceeds the threshold.
- Do not count times at shock where the resource level falls short of the threshold.

Key points:
- $t_0$, $\tau'$, $\tau^*$, $\tau$,
- $D$, $|I| = 1$, $|I| = 0$
Decision problem

Discounted stream of consumption utility up until (random) $\tau$

$$\max_{C(t)} \mathbb{E}_\tau \left[ \int_0^\tau e^{-\rho t} C(t)^{0.5} dt + e^{-\rho \tau} V(R(\tau^+), \tau^+) \right]$$

with stage-2 value:

$$V(R(\tau^+), \tau^+) := \max_{C(t)} \int_{\tau^+}^\infty e^{-\rho t} C(t)^{0.5} dt$$

Subject to:

$$\dot{R}(t) = g(R(t)) - C(t), \ R(0) = R_0$$

$$R(\tau^+) = R(\tau^-) - D(\tau) = \epsilon R(\tau^-)$$

Solve the model by a method developed in Wrzaczek et al. (2020, JOTA)

Resource stock following the shock
Decision problem

\[
\max \mathbb{E}_\tau \left[ \int_0^\tau e^{-\rho t} C(t)^{0.5} \, dt + e^{-\rho \tau} V(R(\tau^+), \tau^+) \right]
\]

with stage-2 value:

\[
V(R(\tau^+), \tau^+) := \max \int_{\tau^+}^{\infty} e^{-\rho t} C(t)^{0.5} \, dt
\]

Subject to:

\[
\dot{R}(t) = g(R(t)) - C(t), \quad R(0) = R_0
\]

\[
R(\tau^+) = R(\tau^-) - D(\tau) = \epsilon R(0)
\]

Discounted stream of consumption utility up until (random) \(\tau\)

Discounted continuation value from \(\tau\)

Discounted flow of consumption stream post-shock, i.e. from \(\tau\) onward

Discounted stream of dividends up until shock

Discounted stream of post-shock dividends

Resource stock following the shock

Solve the model by a method developed in Wrzaczek et al. (2020, JOTA)
Optimal policies in \((R, C)\)-space I

Equilibrium structure:

- stage 2; and stage 1 for \(\epsilon = 1\), i.e. no shock
- stable/high (resilient) and unstable/low (non-resilient) equilibrium (red dots)
- **Skiba threshold** (blue line): Resource level at which the decision-maker is indifferent between the high and low equilibrium

Resilient path: building up resource and consumption over time

Non-resilient path: running down resource at declining levels of consumption
Optimal policies in \((R, C)\)-space II

Stage-1 anticipation of a fully destructive shock \((\epsilon = 0)\):

- shifts high equilibrium downward and low equilibrium and Skiba upward (red curve).
- Additional discounting compromises resilience
Optimal policies in \((R, C)\)-space III

For \(0 \leq \epsilon \leq 1\)…

- …intermediate outcomes with extraction policy…

- …turning more precautionary with increasing \(\epsilon\).
A measure of resilience I

- Resilience at time $t$ given resource stock $R(t)$ (adapted to this model)

\[ \mathcal{R}(R(t), t) = \mathcal{R}_1(R(t), t) + \mathcal{R}_2(R(t), t) \]

lies between 0 = no resilience and 1 = full resilience

- **Ex-ante resilience** (averting the shock): $\mathcal{R}_1(R(t), t)$:
  
  (i) is positive only if the decision-maker follows a resilient path in the first place
  
  (ii) increases in the expected duration of the pre-shock stage 1 (declines in the arrival rate of the shock)

- **Ex-post resilience** (dealing with the shock): $\mathcal{R}_2(R(t), t)$:
  
  (i) increases in the arrival rate of the shock
  
  (ii) increases with the total resilience at the time of each possible (future) shock
A measure of resilience II

- **Ex-ante resilience** (averting the shock)

\[
R_1(R(t), t) = \mathbb{I}_{R(t) \geq R_1^S} \frac{\mathcal{L}(t)}{\mathcal{L}(t)+1}
\]

- where \(\mathbb{I}_{R(t) \geq R_1^S} = 1\) **Resilience** if and only if the resource exceeds the Skiba-threshold \(R_1^S\), and

\(\mathbb{I}_{R(t) < R_1^S} = 0\) **No resilience**.

- and \(\mathcal{L}(t) = \eta^{-1}\) = life-expectancy in stage 1
A measure of resilience III

- **Ex-post resilience** (adapting to the shock)

\[ R_2(R(t), t) = \frac{1}{\mathcal{L}(t) + 1} \int_t^\infty e^{-\eta s} \eta R(s) ds \]

measures resilience to future shocks at \( s \in [t, \infty[ \)

- **Value range**: \( R_i(R(t), t) \in [0,1] \)
  
polar values: 1... full resilience 0... no resilience

Increases in (i) arrival rate of shock; (ii) and in total resilience at any possible time of shock
Resilience of optimal policy

- Benchmark scenario: \( R_0 = 0.2; \rho = 0.1; \eta = 0.5; \epsilon = 0.5 \)
- Resilience **diminishes in discount rate** \( \rho \) and arrival rate \( \eta \) of unavoidable (!) shock \( \eta \) (note that this extends to stage 2 due to reduction in precaution);
- Resilience **increases in initial resource stock** \( R(0) \) and **share of surviving resource stock** \( \epsilon \)
What to do with this measure?

Allows to assign a resilience score to…

- **given** sets of policies => *assessment tool*.
- **Scenarios of optimal decision-making** => explore e.g.
  
  (i) role of discount rate  
  (ii) measures of risk appetite  
  (iii) specific objective function: e.g. corporate vs. welfare oriented policy-maker

- Understand **factors that enhance or hinder resilience** and **incentives** that enhance resilience.
Conclusions

- We **characterise resilience** in a rigorous **decision-theoretic context**
  - (i) Elements: Random shocks and possibility of full system collapse
  - (ii) There is an element of choice in being resilient and surviving

- We provide a **two-part measure of resilience**
  - (i) Resilience and survival in present period (averting shocks)
  - (ii) Resilience following regime change (adapting to shocks)

- We provide a **proof of concept** within a simple model of resource extraction
Outlook I

- Incorporation of **additional features** of resilience:
  
  (i) endogenous hazard and **mitigation**,  
  (ii) endogenous damage (**active protection**),  
  (iii) **adaptation capital** etc.

- Applications of our framework and measure in richer modelling and/or empirical contexts: **climate mitigation, insurance, political resilience**, etc.
Outlook II

- Consider a setting with multiple risks and multiple assets
- Allow variation in impact of each type of shock depending on the type of asset
- Study portfolio allocation depending on information set e.g. about the hazard of each particular shock
Thank you

Questions?

Michael Kuhn
kuhn@iiasa.ac.at